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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, selecting atleast TWO questions from each part.*
2. Use of statistical tables permitted.

PART – A

1 a. Using Taylor's series method, solve $y' = x + y^2$, $y(0) = 1$ at $x = 0.1, 0.2$, considering upto 4^{th} degree term. **(06 Marks)**

b. Using modified Euler's method, find an approximate value of y when x = 0.2 given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0. Take h = 0.1. Perform two iterations in each stage. **(07 Marks)**

c. Using Adams-Bashforth method, obtain the solution of $\frac{dy}{dx} = x - y^2$ at x = 0.8 given that y(0) = 0, y(0.2) = 0.0200, y(0.4) = 0.0795, y(0.6) = 0.1762. Apply the corrector formula twice. **(07 Marks)**

2 a. Employing the Picard's method, obtain the second order approximate solution of the following problem at x = 0.2, $\frac{dy}{dx} = x + yz$, $\frac{dz}{dx} = y + zx$, $y(0) = 1$, $z(0) = -1$. **(06 Marks)**

b. Solve $\frac{dy}{dx} = 1 + xz$ and $\frac{dz}{dx} = -xy$ for x = 0.3 by applying Runge Kutta method given y(0) = 0 and z(0) = 1. Take h = 0.3. **(07 Marks)**

c. Using the Milne's method, obtain an approximate solution at the point x = 0.4 of the problem $\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0$ given that y(0) = 1, y(0.1) = 1.03995, y(0.2) = 1.138036, y(0.3) = 1.29865, y'(0) = 0.1, y'(0.1) = 0.6955, y'(0.2) = 1.258, y'(0.3) = 1.873. **(07 Marks)**

3 a. Define an analytic function and obtain Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Show that $u = e^{2x}$ (x cos2y – y sin2y) is a harmonic function and determine the corresponding analytic function. **(07 Marks)**

c. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

4 a. Evaluate using Cauchy's integral formula $\int_c \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$. **(06 Marks)**

b. Find the bilinear transformation which maps 1, i, -1 to 2, i, -2 respectively. Also find the fixed points of the transformation. **(07 Marks)**

c. Discuss the conformal transformation of $w = z^2$. **(07 Marks)**

PART – B

5 a. Reduce the differential equation:

$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(k^2x^2-n^2)y=0$ into Bessel form and write the complete solution in terms of $\tau_n(x)$ and $\tau_{-n}(x)$. **(06 Marks)**

b. Express $f(x)=x^3+2x^2-x-3$ in terms of Legendre polynomials. **(07 Marks)**

c. If α and β are the roots of $\tau_n(x)=0$ then prove that

$$\int_0^1 x\tau_n(\alpha x)\,\tau_n(\beta x)\,dx=\begin{cases}0, & \alpha\neq\beta\\ \frac{1}{2}[\tau_{n+1}(\alpha)]^2, & \alpha=\beta\end{cases}$$ **(07 Marks)**

6 a. The probability that sushil will solve a problem is 1/4 and the probability that Ram will solve it is 2/3. If sushil and Ram work independently, what is the probability that the problem will be solved by (i) both of them; (ii) at least one of them? **(06 Marks)**

b. A committee consists of 9 students two of which are from first year, three from second year and four from third year. Three students are to be removed at random. What is the chance that (i) the three students belong to different classes; (ii) two belong to the same class and third to the different class; (iii) the three belong to the same class? **(07 Marks)**

c. The contents of three urns are: 1 white, 2 red, 3 green balls, 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. **(07 Marks)**

7 a. The probability mass function of a variate X is

x	0	1	2	3	4	5	6
p(x)	k	3k	5k	7k	9k	11k	13k

i) Find k

ii) Find $p(x<4)$, $p(x\geq 5)$, $p(3<x\leq 6)$, $p(x>1)$

iii) Find the mean. **(06 Marks)**

b. Derive the mean and variance of Poisson distribution. **(07 Marks)**

c. The mean height of 500 students is 151cm and the standard deviation is 15cm. Assuming that the heights are normally distributed, find how many students heights i) lie between 120 and 155cm; ii) more than 155cm. [Given A(2.07) = 0.4808 and A (0.27) = 0.1064, where A(z) is the area under the standard normal curve from 0 to $z>0$]. **(07 Marks)**

8 a. The means of simple samples of sizes 1000 and 2000 are 67.5 and 68.0cm respectively. Can the samples be regarded as drawn from the same population of S.D 2.5cm [Given $z_{0.05}=1.96$]. **(06 Marks)**

b. A random sample of 10 boys had the following I.Q: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q of 100? [Given $t_{0.05}$ for 9d.f = 2.26]. **(07 Marks)**

c. The following table gives the number of aircraft accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week.

Days :	Sun	Mon	Tue	Wed	Thur	Fri	Sat	Total
No. of accidents :	14	16	8	12	11	9	14	84

[Given $\psi^2_{0.05}$ 6d.f = 12.59] **(07 Marks)**

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10ES42

Fourth Semester B.E. Degree Examination, June/July 2016
Microcontrollers

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Distinguish between :
i) Microprocessor and Microcontrollers ii) RISC and CISC Architecture. **(08 Marks)**

b. Briefly discuss the features of 8051 Microcontroller. **(06 Marks)**

c. With the help of a diagram, explain how to interface 8KB EPROM and 8KB RAM, to 8051 Microcontroller. **(06 Marks)**

2 a. Explain the different addressing modes of 8051. Give an example for each one of them. **(08 Marks)**

b. Explain the following instructions :
i) MUL AB ii) DAA iii) MOVC A, @A+DPTR iv) LJMP label **(06 Marks)**

c. What is a stack? Explain with examples, the PUSH and POP instructions. **(06 Marks)**

3 a. What are assembler directives? Explain the functions of the assembler directives DB, EQU, END, ORG. **(06 Marks)**

b. Write an ALP in 8051 to find the largest number among the 14_D, 8 bit number stored in internal RAM. **(07 Marks)**

c. Write an ALP to toggle all bits of P1 every 200ms. Assume that the crystal frequency is 11.0592MHz of 8051. **(07 Marks)**

4 a. Discuss the features of 4 I/O ports of 8051. **(06 Marks)**

b. Interface 4 × 4 keyboard to 8051 and explain how scanning and identifying the key pressed is done. **(07 Marks)**

c. Draw the block diagram to show how 8051 is connected to DAC 0808 at port P_1, using O/P buffer for DAC. Write an 8051 program to generate ramp, signal. **(07 Marks)**

PART – B

5 a. What is the difference between timer and counter? Explain the function of each bit in TMOD. **(04 Marks)**

b. A switch is connected to pin P1.2. write an 8051 C program to monitor SW and create the following frequencies on pin P1.7 SW = 0 : 500 Hz, SW = 1 ; 750 Hz
Use timer 0, mode 1 for both of them. **(08 Marks)**

c. What are external interrupts? Draw the diagrams for activation of external interrupts. How level triggered interrupts are reset? How to set the two external interrupts as edge triggered interrupts? **(08 Marks)**

6 a. Write the steps required for programming 8051 to transfer data serially. **(06 Marks)**

b. Write an 8051 C program to send to two messages "Normal speed" and "High speed" to the serial port. Assuming that SW (switch) is connected to pin P2.0, monitor its status and set the baud rate as follows : SW = 0 ; 28,800 baud rate, SW = 1 ; 56 K baud rate
Assume that XTAL = 11.0592 MHz for both cases. **(08 Marks)**

c. Explain the 4 modes of operation 8255 along with control word format. **(06 Marks)**

7 a. What are the features that make MSP430 suitable for Low-power and portable applications? **(04 Marks)**

b. Explain Registers and peripherals included on chip of MSP430 CPU. **(06 Marks)**

c. Explain the architecture of MSP 430 with a neat diagram. **(10 Marks)**

8 a. Write an assembly program to generate a waveform with ON time of 7 msec and OFF time of 21 msec on P0.5. Assume XTAL of 11.0592MHz. Use timer 0. **(10 Marks)**

b. Explain the bits of SCON register. **(05 Marks)**

c. Draw the Pin diagram of 8255 and briefly explain the signals. **(05 Marks)**

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Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

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10ES43

Fourth Semester B.E. Degree Examination, June/July 2016
Control Systems

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, selecting atleast TWO questions from each part.*

PART – A

1 a. What are the properties of good control system? **(04 Marks)**

b. Construct mathematical model for the mechanical system shown in Fig. Q1(b). Then draw electrical equivalent circuit based on F–V analogy. **(08 Marks)**

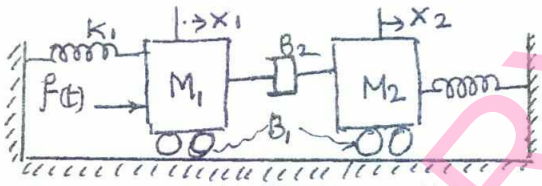

Fig. Q1(b)

c. For electrical system shown in Fig. Q1(C), obtain transfer function $V_2(s)/V_1(s)$. **(08 Marks)**

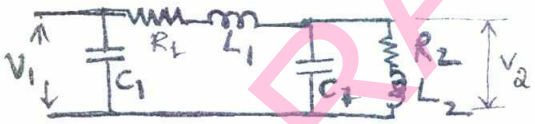

Fig. Q1(c)

2 a. List the features of transfer function. **(04 Marks)**

b. Obtain the transfer function for the block diagram shown in Fig. Q2(b), using block diagram reduction method. **(08 Marks)**

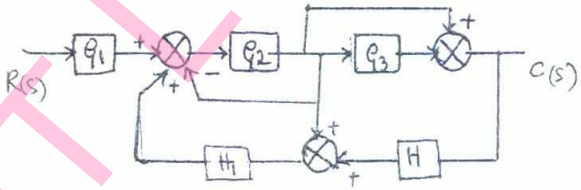

Fig. Q2(b)

c. For the electrical circuit shown in Fig. Q2(c), obtain over all transfer function using Mason's gain formula. **(08 Marks)**

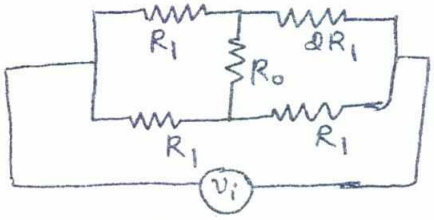

Fig. Q2(c)

3 a. What are static error coefficients? Derive expression for the same. **(06 Marks)**

b. An unity feedback system has $G(s) = \frac{20(1+s)}{s^2(2+s)(4+s)}$, calculate its steady state error co-efficients when the applied input $r(t) = 40 + 2t + 5t^2$. **(06 Marks)**

c. A R–L–C series circuit is an example of second order function. If R = 1 Ω, α = 1H and C = 1F, find response for a step voltage of 10 V connected as input and output across R. **(08 Marks)**

4 a. List the advantages and disadvantages of Routh's criterion (R–H–criterion). **(04 Marks)**

b. A unity feedback control system has $G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$. Using Routh's criterion calculates the range of k for which the system is i) stable ii) has closed loop poles more negative than–1. **(10 Marks)**

c. Find the range of k for which the system, whose characteristic equation is given below is stable. $F(s) = s^3 + (k + 0.5)\, s^2 + 4ks + 50$. **(06 Marks)**

PART – B

5 a. Sketch the root locus for unity feedback having $G(s) = \frac{k(s+1)}{s(s+2)(s^2+2s+2)}$. Determine the range of k for the system stability. **(16 Marks)**

b. Explain how to determine angle of arrival from poles and zeros to complex zeros **(04 Marks)**

6 a. What are the limitations of frequency response methods? **(04 Marks)**

b. A control system having $G(s) = \frac{k(1+0.5s)}{s(1+2s)\left(1+\frac{s}{20}+\frac{s^2}{8}\right)}$ draw bode plot, with k = 4 and find gain margin and phase margin. **(16 Marks)**

7 a. What is polar plot? Explain procedure to sketch polar plot for type 0 and type 1 systems. **(08 Marks)**

b. Sketch the Nyquist plot of a unit feedback control system having the open loop transfer function $G(s) = \frac{5}{s(1-s)}$. Determine the stability of the system using Nyquist stability criterion. **(12 Marks)**

8 a. Find the transfer function for a system having state model as given below :

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad y = [1 \quad 0]x$. **(08 Marks)**

b. Obtain the state model for the electrical system given in Fig. Q8(b) choosing the state variables as $i_1(t)$, $i_2(t)$ and $V_C(t)$. **(12 Marks)**

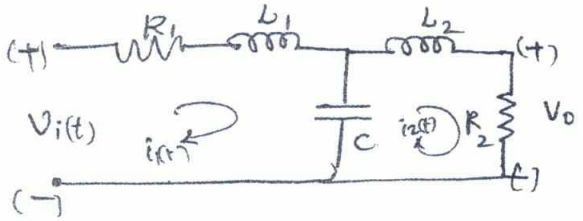

Fig. Q8(b)

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10EE44

Fourth Semester B.E. Degree Examination, June/July 2016
Field Theory

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. State Gauss theorem of electrostatics. List characteristics of Gaussian surface. **(05 Marks)**

b. Determine electric flux density 'D' in Cartesian coordinates caused at p(6, 8, –10) by i) a point charge of 30 mc at origin ii) infinite line charge with $\rho_r = 40$ μc/m ii) A surface charge with $\rho_s = 57.2$ μc/m^2 on a plane z = – 9m. **(08 Marks)**

c. Evaluate both side of divergence theorem for the region r ≤ a (spherical coordinates) having flux density $D = \frac{5r}{3} a_r$ c/m^2 **(07 Marks)**

2 a. Prove that : $E = -\nabla V$ **(05 Marks)**

b. Determine work done in carrying a charge of –2C from (2, 1, –1) to (8, 2, –1) in an electric field $E = ya_x + xa_y$ v/m along the path $x = 2y^2$. **(07 Marks)**

c. Three point charges 3 coul, 4 coul and 5 coul are to be situated at corner of an equilateral triangle of side 5 m. Find energy density at the centre of triangle. **(08 Marks)**

3 a. Derive Poisson's and Laplace equation. **(06 Marks)**

b. A potential field is given by $v = x^2yz + Ay^3z$ volts determine of 'A' such that v satisfies Laplace equation and hence find electric field E at p(2, 1, –1). **(06 Marks)**

c. A spherical capacitor has a capacitance of 54 pF. It consists of two concentric spheres with inner and outer radii differing by 4 cm. Dielectric in between is air. Determine inner and outer radii. **(08 Marks)**

4 a. State and explain Ampere's circuital law. **(05 Marks)**

b. Determine magnetic flux density 'B' at 'P' for a current loop shown in Fig.Q4(b). **(09 Marks)**

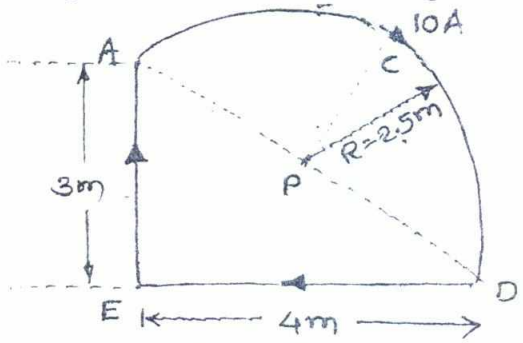

Fig. Q4(b)

c. Clearly distinguish between scalar magnetic potential and vector magnetic potential. **(06 Marks)**

PART – B

5 a. Derive Lorentz force equation for a moving change placed in a combined electric and magnetic field. **(06 Marks)**

b. A point charge Q = 18 nc moves with a velocity of 5×10^6 m/sec in the direction of $0.06a_x + 0.75a_y + 0.3a_z$. Determine magnitude of force experienced by the charge when placed in i) electric field $E = -3a_x + 4a_y + 6a_z$ kv/m ii) magnetic field $E = -3a_x + 4a_y + 6a_z$ mT iii) combined E and B. **(08 Marks)**

c. An air cored toroid has a cross sectional area of 6 cm^2, a mean radius of 15 cm and is wound with 500 turns and carries a current of 4A. Find the magnetic field intensity at the mean radius. **(06 Marks)**

6 a. Explain Faraday's laws applied to : i) stationary path, changing field and ii) steady field, moving circuit. **(06 Marks)**

b. List Maxwell's equations for both : i) steady and ii) Time varying fields in differential and integral form, also mention the relevant laws they demonstrate. **(08 Marks)**

c. A straight conductor of length 0.2m, lies on x-axis with one end at origin. The conductor is subjected to a magnetic flux density $B = 0.04a_y$ Tesla and the velocity $v = 2.5 \sin 10^3 t\, a_z$ m/sec. Determine motional emf induced in the conductor. **(06 Marks)**

7 a. Derive wave equation for E in a general medium. **(06 Marks)**

b. State and explain Poynting theorem. **(06 Marks)**

c. A lossless dielectric medium has $\sigma = 0$, $\mu_r = 1$ $\varepsilon_r = 1$. A electromagnetic wave has field as $H = -0.1 \cos(\omega t - z)a_x + 0.5 \sin(\omega t - z)a_y$ A/m. Find : i) phase constant, ii) angular velocity iii) the wave impedance iv) components of electric field intensity of the wave. **(08 Marks)**

8 a. Derive an expression for transmission coefficient and reflection coefficient and relate them. **(08 Marks)**

b. Define standing wave ratio. Write an expression for it. **(04 Marks)**

c. Determine the amplitude of reflected and transmitted 'E' and 'H' at the interface between two regions. Characteristics of region 1 are $\varepsilon_{r_1} = 8$, $\mu_{r1} = 0$; $\sigma_1 = 0$ and region 2 is free space. The incident E_0^i in region 1 is of 1.5 V/m. Assume normal incidence. Also find average power in two regions. **(08 Marks)**

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10EE45

Fourth Semester B.E. Degree Examination, June/July 2016
Power Electronics

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. Draw the control characteristics of the following: i) SCR; ii) BJT; iii) MCT; iv) GTO; v) MOSFET. **(10 Marks)**

b. Explain any four industrial applications of power electronics. **(10 Marks)**

2 a. What is a MOSFET? Draw static and switching characteristics of a MOSFET and explain the operation of MOSFET as a switch. **(12 Marks)**

b. A simple transistor switch is used connect a 24V DC supply across a relay coil, which has a DC resistance of 200Ω. An input pulse of 0 to 5V amplitude is applied through a series base resistance R_B at the base so as to turn on transistor switch. Calculate,
i) ICS ; ii) Value of resistor R_B, required to obtain over drive factor of 2; iii) Total power dissipation in the transistor that occurs during the saturation state. **(08 Marks)**

3 a. With neat circuit diagram and waveforms explain R-triggering technique of SCR. **(08 Marks)**

b. Derive an expression for the anode current of thyristor with the help of two transistor analogy. **(08 Marks)**

c. An SCR is connected in series with a 0.5H inductor and 20Ω resistance. A 100V DC voltage is applied to this circuit. If the latching current of the SCR is 4mA, find the minimum width of the gate trigger pulse required to properly turn-on the SCR. **(04 Marks)**

4 a. What do you mean by natural and forced commutation in thyristor circuits? Explain with waveform and circuit complimentary commutation. **(12 Marks)**

b. Explain external pulse commutation with neat circuit and waveform and derive an expression for circuit turn off time. **(08 Marks)**

PART – B

5 a. What is controlled rectifier? With neat circuit and waveform explain single phase semiconverter with R-load. **(12 Marks)**

b. For a 3ϕ fully controlled SCR bridge converter operating from 400V, 3ph ac supply, calculate the average DC output voltage for a firing angle of 45°. Write the necessary diagram. **(08 Marks)**

6 a. Explain the classification of chopper with neat circuit diagram and wave form. **(12 Marks)**

b. A step-up chopper has input voltage of 220 V and an output voltage of 660 V. If the non-conducting time of thyristor chopper is 100μsec, compute the pulse width of output voltage. In case pulse width is halved for constant frequency operation, find the new output voltage. **(08 Marks)**

7 a. What is an inverter? Explain half bridge inverter with R-load. **(10 Marks)**

b. Briefly explain performance parameters of an inverter. **(06 Marks)**

c. With relevant waveform explain the sinusoidal pulse width modulation of an inverter. **(04 Marks)**

8 a. With the help of neat circuit diagram and waveform explain on-off control and derive an expression for i) RMS value of o/p voltage; ii) Input power factor. **(12 Marks)**

b. A single phase half wave ac voltage controller has a resistive load of R = 5Ω and input voltage V_S = 120V, 60Hz. The delay angle of thyristor is $\alpha = \pi/3$. Determine: i) RMS output voltage; ii) Input power factor; iii) Average input current. **(08 Marks)**

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10EE46

Fourth Semester B.E. Degree Examination, June/July 2016
Transformers and Induction Machines

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. With neat schematic diagram, explain core and shell type transformer. **(06 Marks)**

b. Discuss the working condition of 1-ϕ transformer on resistive load with vector diagram. **(06 Marks)**

c. A 230/460 volts transformer has a primary resistance of 0.2Ω and a reactance of 0.5Ω and the corresponding values for the secondary are 0.75Ω and 1.8Ω respectively. Find the secondary terminal voltage when supplying i) 10A at 0.8 p.f. lagging; ii) 10A at 0.8 p.f leading. **(08 Marks)**

2 a. Develop the equivalent of a 1-ϕ transformer referred to primary side from the fundamentals. **(06 Marks)**

b. Discuss the different types of losses in transformer and derive efficiency of transformer. **(06 Marks)**

c. A 200 kVA single phase transformer is in circuit continuously. For 8 hours in a day the load is 80 kW at unity power factor and for the remaining period of 24 ours it runs on no-load. Full load copper losses are 3.02 kW and the iron losses are 1.6 kW. Find all day efficiency. **(08 Marks)**

3 a. Discuss the essential and desirable conditions to be fulfilled for operating two single phase transformers in parallel. **(06 Marks)**

b. What is an auto transformer? State its merits and demerits over two winding transformer. **(06 Marks)**

c. Two transformers have following characteristics:
Transformer 1 — % IR = 1.0% and % IX = 5.0%
Transformer 2 — % IR = 1.5% and % IX = 4.0%
How they will share a load of 100 kVA at 0.8 p.f lagging? **(08 Marks)**

4 a. Draw the soft connection of transformer and mark the terminals. Explain its merits and demerits. **(06 Marks)**

b. Briefly discuss the choice of transformer connections. **(06 Marks)**

c. Give the detail analysis of load sharing between two three phase transformers operating in parallel. **(08 Marks)**

PART – B

5 a. Bring out clearly, with the help of neat sketches the difference between the 3-phase slip ring induction motor and three phase squirrel cage induction motor. **(06 Marks)**

b. Explain the terms slip, slip frequency and give the relation between them. **(06 Marks)**

c. A 746 kW, 3-phase, 50 Hz 16-pole induction motor has a rotor impedance of (0.02 + j0.15) Ω at stand still full-load torque is obtained at 360 rpm. Calculate: i) The ratio of maximum to full-load torque; ii) The speed for maximum torque and iii) The rotor resistance to be added to get maximum starting torque. **(08 Marks)**

6 a. Enumerate various components of power loss in an induction motor and name the parts where in these occur. **(06 Marks)**

b. Draw the circle diagram of a 3-phase mesh connected, 30 h.p, 500V, 4 pole, 50Hz cage type induction motor. The table gives the measurements of line current and voltage and readings of two watt meters connected to measure the input power.

No-load	500V	8.3A	+2.85 kW	-1.35 kW
Blocked rotor test	100 V	32 A	-0.75 kW	+2.35 kW

Find from the diagram for full-load:

i) Line current
ii) Power factor
iii) Efficiency and
iv) Maximum output. **(14 Marks)**

7 a. With neat sketch, explain the construction of deep bar cage rotor motor. **(06 Marks)**

b. Draw the torque-speed characteristics double cage rotor motor. How these characteristics are different from squirrel cage induction motor? **(06 Marks)**

c. Describe with a neat diagram the principle of operation of induction generator. **(08 Marks)**

8 a. Why the starter is necessary to start an induction motor? Mention the various methods of starting and discuss the limitations of these methods. **(06 Marks)**

b. With neat schematic diagram, explain the method of star-delta transformer. **(06 Marks)**

c. With neat sketches, explain the construction working principle of
i) Split phase and ii) Capacitor start single phase induction motor **(08 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2016
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(07 Marks)**

b. Prove that the general equation of first degree in x, y, z represents a plane. **(07 Marks)**

c. Find the angle between the lines,

$$\frac{x-1}{1}=\frac{y-5}{0}=\frac{z+1}{5} \text{ and } \frac{x+3}{3}=\frac{y}{5}=\frac{z-5}{2}.$$ **(06 Marks)**

2 a. Prove that the lines,

$$\frac{x-5}{3}=\frac{y-1}{1}=\frac{z-5}{-2} \text{ and } \frac{x+3}{1}=\frac{y-5}{3}=\frac{z}{5} \text{ are perpendicular.}$$ **(07 Marks)**

b. Find the shortest distance between the lines.

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}.$$ **(07 Marks)**

c. Find the equation of the plane containing the point (2, 1, 1) and the line,

$$\frac{x+1}{2}=\frac{y-2}{3}=\frac{z+1}{-1}$$ **(06 Marks)**

3 a. Find the constant 'a' so that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are co-planar. **(07 Marks)**

b. If $\vec{a}=2\hat{i}+3\hat{j}-4\hat{k}$ and $\vec{b}=8\hat{i}-4\hat{j}+\hat{k}$ then prove that $\vec{a}$ is perpendicular to $\vec{b}$ and also find $\left|\vec{a}\times\vec{b}\right|$. **(07 Marks)**

c. Find the volume of the parallelopiped whose co-terminal edges are represented by the vectors,

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ **(06 Marks)**

4 a. Find the velocity and acceleration of a particle moves along the curve $\vec{r}=e^{-2t}\hat{i}+2\cos 5t\hat{j}+5\sin 2t\hat{k}$ at any time 't'. **(07 Marks)**

b. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i}+\hat{j}+2\hat{k}$. **(07 Marks)**

c. Find the divergence of the vector $\vec{F}=(xyz+y^2z)\hat{i}+(3x^2+y^2z)\hat{j}+(xz^2-y^2z)\hat{k}$. **(06 Marks)**

5 a. $\vec{F}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, show that $\vec{F}\cdot\text{curl}\,\vec{F}=0$. **(07 Marks)**

b. Show that the vector field, $\vec{F}=(3x+3y+4z)\hat{i}+(x-2y+3z)\hat{j}+(3x+2y-z)\hat{k}$ is solenoidal. **(07 Marks)**

c. Find the constants a, b, c such that the vector field,

$\vec{F}=(x+y+az)\hat{i}+(x+cy+2z)\hat{j}+(bx+2y-z)\hat{k}$ is irrotational. **(06 Marks)**

6 a. Prove that $L(\sin at) = \frac{a}{s^2 + a^2}$. **(07 Marks)**

b. Find $L[\sin t \quad \sin 2t \quad \sin 3t]$. **(07 Marks)**

c. Find $L[\cos^3 t]$. **(06 Marks)**

7 a. Find the inverse Laplace transform of $\frac{1}{(s+1)(s+2)(s+3)}$. **(07 Marks)**

b. Find $L^{-1}\left[\log\left(1+\frac{a^2}{s^2}\right)\right]$. **(07 Marks)**

c. Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$. **(06 Marks)**

8 a. Solve the differential equation, $y'' + 2y' + y = 6te^{-t}$ under the conditions $y(0) = 0 = y'(0)$ by Laplace transform techniques. **(10 Marks)**

b. Solve the differential equation, $y'' - 3y' + 2y = 0 \quad y(0) = 0, \quad y'(0) = 1$ by Laplace transform techniques. **(10 Marks)**

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